Simulation of conjugate heat transfer between fluid-saturated porous media and solid wall

Sheng Chen *[1,2],

1. Institute for Modelling and Simulation in Fluodynamics, Nanoscience and Industrial Mathematics "Gregorio Millán Barbany", Universidad Carlos III de Madrid, Leganes 28911, Spain

2. Faculty of Engineering, The University of Nottingham, University Park, Nottingham NG7 2RD, UK

** Corresponding author. E-mail address: ezzsc2@exmail.nottingham.ac.uk*

Abstract

Conjugate heat transfer between fluid-saturated porous media and solid walls is an important issue in many disciplines. For traditional numerical techniques, it is still a great challenge to treat conjugate problems, especially with complicated interfaces. In the present work, a new numerical approach, based on the lattice Boltzmann (LB) method, is proposed to address such challenge. In the present approach no explicit special treatment on coupled interfaces between fluid-saturated porous media and solid walls is required. Moreover, no additional source term, which exists in the available LB models, is required. It can guarantee that the simplicity, accuracy and stability of the present model are better than those models. The accuracy and re-

liability of the present approach are validated by three nontrivial benchmark tests. The excellent agreement with previous published data demonstrates its feasibility and simplicity for modelling conjugate heat transfer with complicated interfaces between fluid-saturated porous media and solid walls. The present work takes a single-relaxation-time model for example to address conjugate heat transfer problems and its multiple-relaxation-time counterpart can be established straightforwardly in the same way.

Key words: Lattice Boltzmann method; conjugate heat transfer; porous media

1 Introduction

Conjugate heat transfer between fluid-saturated porous media and solid walls is an important issue in many disciplines, such as environment and building engineering, energy engineering and biomedical engineering, since in many applications of these disciplines fluid-saturated porous media and solid walls co-exist[1–3]. At the beginning, conjugate heat transfer problems were solved by some (semi)empirical criteria as then no other available tool [4]. With the rapid development of computer science, modern conjugate heat transfer models, based on strictly mathematical descriptions, have been established to replace the (semi)empirical analogies [5]. Originally, conjugate heat transfer was numerically treated as a coupling type of the Dirichlet and Neumann boundary conditions [6]. By such treatment, a compatibility condition should be paid high attention to. Later, it was found a Robin boundary condition

treatment may be more useful for some conjugate heat transfer situations [7]. The accuracy and stability of these numerical treatments have been compared recently [8]. One of the keys to affect the accuracy of conjugate heat transfer modelling is the quality of the mesh [9]. However, it was found that even unstructured grids could not meet a extremely complex interface, such as an automobile engine compartment. To address this challenge, many advanced modern techniques, including immersed boundary method [10] and fictitious-domain method [11], are introduced for conjugate heat transfer simulation. So far, numerical simulation has achieved great success in many advanced engineering applications, such as simulating conjugate heat transfer of continuously moving surfaces [12], of compound material [13] and of turbulent forced convection in channels [14]. A comprehensive latest review on this topic has been presented by Dorfman and Renner [5]. Nowadays the fine structures of conjugate heat transfer between fluid-saturated porous media and solid walls also can be predicted exactly with the aid of solving the conjugate heat transfer models numerically. The available literature on numerically modelling conjugate heat transfer between fluid-saturated porous media and solid walls may fall into two categories: (1) solid walls embedded in fluid-saturated porous media and (2) fluid-saturated porous media bounded by solid walls. The industrial background of the former category is heat transfer enhancement of a fin [15,16], while the latter is a popularly adopted research prototype for solar collectors, energy storage systems filled by phase-change materials and so on [17–19]. A latest review on this topic was presented in Ref.[3]. For engineering

research, usually fluid-saturated porous media are modelled at the REV (representative elementary volume) scale (e.g. Refs. [15–19]), due to the balance between necessary macroscopic information and computational cost [20].

Although solving conjugate heat transfer problems numerically is a great breakthrough in thermal science and engineering, it is still a great challenge for the current state-of-art numerical techniques to address conjugate heat transfer problems with complicated interfaces [3,9]. The difficulty stems from the fact that, for the conventional numerical approaches, specific treatments are required to guarantee the continuity of temperature profile and heat flux across the interfaces between fluid-saturated porous media and solid walls [2,3,15–19].

During the past three decades, the lattice Boltzmann (LB) method has attracted increasing attention due to its some intrinsic advantages, such as relatively easy treatment of complicated geometry, high parallel computing efficiency and capturing interaction between different phases at a mesoscopic level [21]. Until now, the LB method has been widely used to investigate heat and mass transfer in porous media[22–25]. Especially, as it is a particle-based numerical solver, the LB method can guarantee, automatically, the continuity of a certain macroscopic quantity and of its flux across an arbitrary interface within the investigated domain, if the macroscopic quantity and its flux can be recovered from the zeroth- and first-order moment of the corresponding pseudo-particle distribution function, respectively. As pointed out by our

recent publication[26], this feature may provide a great advantage over the traditional numerical methods for conjugate heat transfer research as in the LB framework one need not explicitly treat the topology of the interface where the conjugated boundary condition should be strictly satisfied [15–19] . So far there have been a number of efforts to model conjugate heat transfer between fluid flows and solid walls by the LB method [27–32]. Recently, the present authors compared the advantages and disadvantages of the available models and then proposed a new LB conjugate model to remedy their drawbacks [33]. However, the efforts to extend the LB method for conjugate heat transfer between fluid-saturated porous media and solid walls at the REV scale are quite sparse. Recently, Gao et al. [34] designed a LB model to simulate conjugate heat transfer in porous media. In their model, an additional source term is added into their LB evolving equation to guarantee to recover the macroscopic energy equation correctly. Unfortunately, their source term includes temporal difference operation, which will hamper numerical stability, code simplicity as well as order of accuracy, as discussed in our work [33].

The purpose of the present work is to bridge the above gap since an efficient REV scale model for conjugate heat transfer between fluid-saturated porous media and solid walls is very crucial in a lot of practical applications, such as solidification processes, modern building thermal insulators and cooling hot intrusions in a geological setting [1,3]. The present work can extend the LB method to these important areas and accelerate the corresponding research, e-

specially where complicated interfaces exist. In many popularly used numerical techniques, such as the finite volume and finite element approaches, a special treatment is required to guarantee the heat flux to be continuous across the interface between two domains with different thermophysical properties [5]. Such treatment will become extremely complicated if the topology of the interface becomes complex, for example irregularly curved interfaces [9]. As stressed in a recent brief review [9], to simulation conjugate heat transfer problems with complex interfaces is still a significant challenge even for the mature numerical techniques. Fortunately, in the LB framework such treatment can be avoided [33]. Compared with previous models, such as Gao's LB model[34] and Chen's LB model [33] for conjugate problems, there is no additional source term in the present scheme, which can guarantee that simplicity, accuracy and stability of the present model are better. The rest of the present paper is organized as follow. In Section2, the macroscopic governing equations for conjugate heat transfer between fluid-saturated porous media and solid walls, at the REV scale, are presented. Then we will show how to establish a simple numerical approach, based on the LB method, to model such conjugate heat transfer problems. In the present approach, the complexity to explicitly treat the interfaces between fluid-saturated porous media and solid walls, which is a great challenge for the popularly used numerical methods [1–3,15–19], can be avoided. Numerical validation for the present model is conducted in Section 4, followed by a conclusion on this work. What should be emphasized is although in the present work only a single-relaxation-time LB model is presented, its

multiple-time-relaxation counterpart can be constructed in the same way, for example based on the MRT REV scale porous model by Liu et al. [35].

2 Governing equations for conjugate heat transfer between fluid-saturated porous media and solid walls

The macroscopic governing equation for heat transfer in fluid-saturated porous media, at the REV scale, reads [1,2,36]:

$$\sigma \partial_t T + \nabla_\alpha T u_\alpha = \nabla_\alpha \kappa_m \nabla_\alpha T. \tag{1}$$

where u_α and T are the volume-averaged velocity and temperature of fluid in the saturated porous media, respectively. The parameter $\sigma = \varepsilon + (1-\varepsilon)C_s/C_f$, where ε is the porosity of the porous media, and C_s and C_f are the heat capacitance (the product of density and heat capacity) of solid porous matrix and of saturating fluid, respectively. In addition, $\kappa_m = \lambda_m/C_f$ is the effective thermal diffusivity of fluid-saturated porous media, where λ_m is the corresponding effective thermal conductivity. Bear in mind Eq.(1) is valid only when solid porous matrix and saturating fluid meet the thermal equilibrium restriction. If not, the governing equations for heat transfer in porous media should be modified to consider the thermal non-equilibrium effect [37]. For thermal non-equilibrium cases, besides Eq.(1), meanwhile an additional heat conduction equation for solid porous matrix should be solved and Gao et al. developed a

LB model for such cases [37]. The method proposed in the present paper to deal with conjugate heat transfer between fluid-saturated porous media and solid wall will not be affected by the alteration of the governing equations for heat transfer in porous media, so the present method can still work well for simulating conjugate heat transfer between fluid-saturated porous media and solid wall considering the thermal non-equilibrium effect if Gao's LB model [37] is adopted to replace Guo's LB model [36] used in this work.

For solid walls, the macroscopic governing equation for temperature field reads [1–3]:

$$\partial_t T = \nabla_\alpha \kappa_s \nabla_\alpha T. \tag{2}$$

where $\kappa_s = \lambda_s / C_s$ is the thermal diffusivity of solid walls, where λ_s and C_s are the thermal conductivity and heat capacitance of solid walls.

For conjugate heat transfer scenarios, on the interfaces between fluid-saturated porous media and solid walls, the following restrictions should be satisfied [1–3]:

$$T_+ = T_- \tag{3}$$

$$n_\alpha [\lambda_m \nabla_\alpha T]_+ = n_\alpha [\lambda_s \nabla_\alpha T]_- \tag{4}$$

where n_α is normal to the interface, and $[\quad]_+$ and $[\quad]_-$ indicate the parameters at each side of the interface. For the popularly used numerical methods, specific treatments are required to ensure Eqs. (3)-(4) to be held exactly on the interfaces [2,3,15–19]. Unfortunately, for complicated interfaces, it is a great

challenge (sometimes impossible) to identify the normal direction n_α [9,29]. For example, as shown by the Table I and II in Ref.[29], there is an extremely complex extrapolation process to identify the normal direction and to evaluate the thermal flux on an irregular interface.

3 LB model for conjugate heat transfer between fluid-saturated porous media and solid walls

3.1 LB evolving equation

The LB model for REV scale heat transfer in fluid-saturated porous media, proposed by Guo et al. [36], has been popularly used in the research area of porous media. The macroscopic governing equation Eq.(1) can be recovered from Guo's model. However, as pointed out in our previous work [26], their LB model can not satisfy the continuity of heat flux across the interfaces, namely Eq.(4), since the variation of heat capacitance across the interfaces between fluid-saturated porous media and solid walls can not be treated in their REV scale model appropriately. Consequently, conjugate heat transfer is beyond the capability of Guo's model.

In order to address this challenge, we construct a new evolving equation which reads

$$g_j(x_\alpha + e_{\alpha,j}\Delta t, t + \Delta t) - g_j(x_\alpha, t) = -\tau_T^{-1}[g_j(x_\alpha, t) - g_j^{(eq)}(x_\alpha, t)]. \qquad (5)$$

In Eq.(5) τ_T is the dimensionless relaxation time for temperature field simulation. What should be stressed is that Eq.(5) have no real physics interpretation but just a numerical solver for macroscopic governing equation of heat transfer Eq.(1).

The equilibrium distribution in Eq.(5) reads

$$g_j^{(eq)} = \begin{cases} T(\eta C - C_0) + \omega_j TC(\frac{C_0}{C} + \frac{e_{j\alpha}u_\alpha}{c_s^2}), & j = 0 \\ \omega_j TC(\frac{C_0}{C} + \frac{e_{j\alpha}u_\alpha}{c_s^2}), & j \neq 0 \end{cases} \qquad (6)$$

where ω_j represents the weight coefficients and e_j denotes the discrete velocity direction [36]. The parameter c_s satisfies $c_s^2\delta_{\alpha\beta} = \sum_j \omega_j e_{j\alpha}e_{j\beta}$. C_0 is a reference value of heat capacitance and the heat capacitance C satisfies the following rules

$$C = \begin{cases} C_f, \quad in \quad porous \quad media \\ C_s, \quad in \quad solid \quad wall \end{cases} \qquad (7)$$

Here an explanation should be made for Eq.(7): the governing equation for temperature field Eq.(1) has been normalized by C_f, so within the porous media $C = C_f$, which results from the thermal equilibrium assumption. If porous media are under a thermal non-equilibrium condition, C will be set

as the heat capacitance of the saturating fluid or of the solid porous matrix, respectively.

In addition, the parameter η satisfies

$$\eta = \begin{cases} \sigma, & in \quad porous \quad media \\ \\ 1, & in \quad solid \quad wall \end{cases} \tag{8}$$

The temperature T is obtained by

$$T = \frac{\sum\limits_{j} g_j}{\eta C}. \tag{9}$$

and the thermal conductivity λ is given by

$$\lambda = (\tau_T - 1/2) c_s^2 \Delta t C_0. \tag{10}$$

where

$$\lambda = \begin{cases} \lambda_m, & in \quad porous \quad media \\ \\ \lambda_s, & in \quad solid \quad wall \end{cases} \tag{11}$$

The sub-domain occupied by fluid-saturated porous media and that occupied by solid walls share a common LB evolving equation Eq.(5). As shown above, in the present model, we need not explicitly treat the interfaces between

fluid-saturated porous media and solid walls which the traditional numerical approaches have to face [3,9] (namely, to ensure Eqs. (3)-(4) to be exactly held on the interfaces by some complicated special numerical treatments).

3.2 Multiscale expansion and recovered macroscopic equations

Equation (5) can be expanded in Taylor series as [21]

$$\Delta t(\partial_t + e_{j\alpha}\nabla_\alpha)g_j + \frac{\Delta t^2}{2}(\partial_t + e_{j\alpha}\nabla_\alpha)^2 g_j + \frac{1}{\tau_T}[g_j - g_j^{(eq)}] = O(\Delta t^3). \quad (12)$$

Introducing the multiscale expansion $\partial_t = \epsilon\partial_{t1} + \epsilon^2\partial_{t2}$, $\nabla_\alpha = \epsilon\nabla_{\alpha 1}$ and $g_j = g_j^{(eq)} + \epsilon g_j^{(1)} + \epsilon^2 g_j^{(2)} + O(\epsilon^3)$, where $\epsilon \ll 1$ [36], we can sort Eq. (12) in terms of ϵ and ϵ^2 as

$$(\partial_{t1} + e_{j\alpha}\nabla_{\alpha 1})g_j^{(eq)} = -\frac{g_j^{(1)}}{\Delta t\tau_T} + O(\epsilon). \quad (13)$$

$$\partial_{t2}g_j^{(eq)} + (\partial_{t1} + e_{j\alpha}\nabla_{\alpha 1})[(1 - \frac{1}{2\tau_T})g_j^{(1)}] = -\frac{g_j^{(2)}}{\Delta t\tau_T} + O(\epsilon^2). \quad (14)$$

With the symmetry properties of the lattice $\sum_j \omega_j e_{j\alpha} = 0$ and $\sum_j \omega_j e_{j\alpha}e_{j\beta} = c_s^2\delta_{\alpha\beta}$ we can obtain

$$\sum_j g^{(eq)} = \eta CT, \quad (15)$$

$$\sum_j e_{j\alpha}g^{(eq)} = CTu_\alpha, \quad (16)$$

$$\sum_j e_{j\alpha}e_{j\beta}g^{(eq)} = C_0 Tc_s^2\delta_{\alpha\beta}. \quad (17)$$

With the aid of Eqs.(15)-(17), as well as $\sum_j g_j^{(1)} = \sum_j g_j^{(2)} = 0$, the summation

of Eqs.(13)-(14) over the discrete direction $e_{j\alpha}$ reads

$$\partial_{t1}\eta CT + \nabla_{\alpha 1} CTu_\alpha = 0 + O(\epsilon), \tag{18}$$

$$\partial_{t2}\eta CT + \nabla_{\alpha 1}[c_s^2(\frac{1}{2} - \tau_T)\Delta t \nabla_{\alpha 1} C_0 T] = 0 + O(\epsilon^2). \tag{19}$$

Because C_0 is a constant across the whole investigated domain, $\nabla_{\alpha 1} C_0 T = C_0 \nabla_{\alpha 1} T$. Accordingly Eq.(19) can be re-written as

$$\partial_{t2}\eta CT + \nabla_{\alpha 1}[c_s^2(\frac{1}{2} - \tau_T)\Delta t C_0 \nabla_{\alpha 1} T] = 0 + O(\epsilon^2). \tag{20}$$

Combining Eqs.(18) and (20), we can obtain the final recovered macroscopic governing equation for temperature field

$$\partial_t \eta CT + \nabla_\alpha CTu_\alpha = \nabla_\alpha \lambda \nabla_\alpha T + O(\epsilon^2). \tag{21}$$

where $\lambda = c_s^2(\tau_T - \frac{1}{2})\Delta t C_0$.

Introducing Eqs.(7),(8) and (11), Eq.(21) becomes

$$\partial_t \sigma C_f T + \nabla_\alpha C_f T u_\alpha = \nabla_\alpha \lambda_m \nabla_\alpha T \tag{22}$$

within the fluid-saturated porous media sub-domain and

$$\partial_t C_s T = \nabla_\alpha \lambda_s \nabla_\alpha T \tag{23}$$

within the solid wall sub-domain (for a stationary solid wall $u_\alpha = 0$).

Eqs.(22)-(23) are the conservation form of Eqs.(1)-(2), respectively. As pointed out by Karani et al. [32] for conjugate heat transfer research the conservation form of energy equation was better to guarantee the continuity of heat flux across interfaces. Consequently, the present model can automatically recover Eq.(1) (temperature governing equation of fluid-saturated porous media) and Eq.(2) (temperature governing equation of solid walls) exactly. Moreover, as demonstrated in our previous work [26], since the thermal conductivity λ depends on a constant C_0, rather than C, therefore the continuity of heat flux across the interfaces between fluid-saturated porous media and solid walls can be guaranteed automatically in the present framework, without any explicit conjugate interface treatment that is unavoidable for traditional numerical techniques [1–3,15–19].

4 Numerical validation

In order to validate the present model, three simple but non-trivial benchmark tests are adopted. The first one is conjugate heat conduction between a stratified saturated porous layer and a solid wall. Analytical solution is available for this benchmark so it can be used to check the accuracy of the present model. The second one is conjugate natural convection in a square fluid-saturated porous cavity investigated in Ref.[2], which is an important research proto-

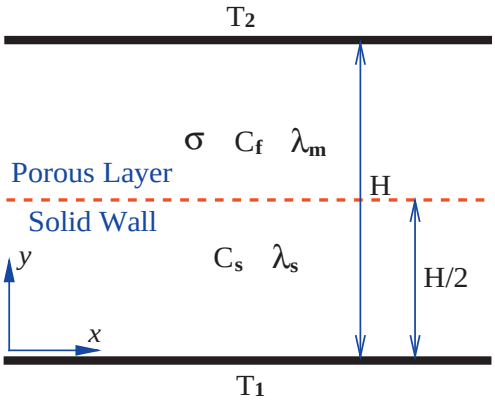


Fig. 1. Schematic configuration of conjugate heat conduction between a stratified saturated porous layer and a solid wall.

type for various applications. The last one is conjugate natural convection in a fluid-saturated porous cavity with a hot triangular thick wall [3], which can demonstrate the simplicity and effectiveness of the present model for complicated conjugate interfaces. The choice of validation benchmark with simple configuration can avoid unexpected numerical errors which will hamper the assessment for a new numerical approach, so for all recent open work of developing new LB models for conjugate heat transfer simulation and porous media modelling [22-37], the test cases adopted for numerical validation are characterized by simple configuration. For conjugate natural convection, fluid flow should be modelled simultaneously. In the present work, the LB model developed in Ref.[36] is adopted for flow field simulation and the boundary scheme proposed in our previous work [38] is employed here for flow boundaries.

4.1 Conjugate heat conduction between a stratified saturated porous layer and a solid wall

Figure 1 illustrates the schematic configuration of conjugate heat conduction between a stratified saturated porous layer and a solid wall. The temperature on the top surface of the porous layer is T_2 and that on the bottom surface of the solid wall is T_1. The periodic boundary condition is adopted for the left and right side of the domain. Here we set $T_2/T_1 = 2$, $\sigma = 1$, $C_s = 1.5C_f$ and $\lambda_m = 3\lambda_s$. In addition, $C_0 = (C_f + C_s)/2$ and a grid solution 60×60 is employed. The corresponding analytic solution of temperature profile reads [26]

$$T(y) = \begin{cases} \frac{3y}{2H}(T_2 - T_1) + T_1, & 0 \le y \le 0.5H. \\ \\ (\frac{y}{2H} + 0.5)(T_2 - T_1) + T_1, & 0.5H \le y \le H. \end{cases}$$

Figure 2 illustrates the numerical data obtain by the present model, compared with the analytic solution. The present numerical results agree well with the analytical solution.

4.2 Conjugate natural convection in a square fluid-saturated porous cavity

Figure 3 illustrates the configuration of conjugate natural convection in a square fluid-saturated porous cavity, which is the same as that in Ref. [2]. There is a vertical interface between the solid wall and the porous cavity. The

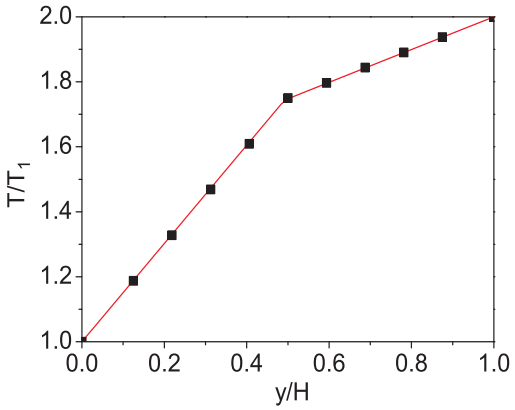


Fig. 2. Temperature profile of conjugate heat conduction between a stratified saturated porous layer and a solid wall: scatters-analytic solution, solid line-numerical data.

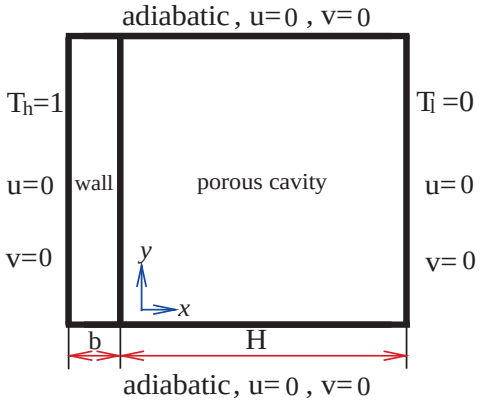
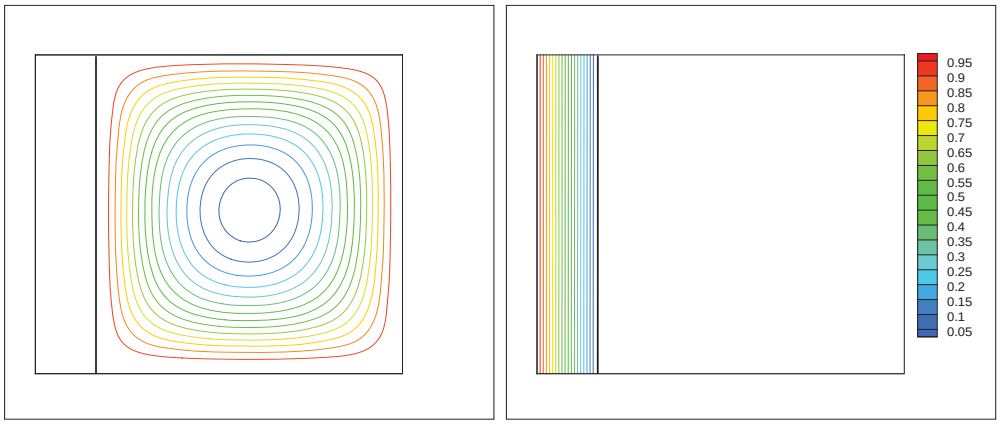


Fig. 3. Schematic configuration of conjugate natural convection in a square fluid-saturated porous cavity.

parameters used in the present research are $b/H = 0.2$, $C_s = C_f = 1$, $\sigma = 1$, $\varepsilon = 0.9$, the Darcy number $Da = 10^{-3}$, the Prandtl number $Pr = 1$, the Rayleigh number $Ra = 10^5$, $\lambda_m/\lambda_f = \varepsilon + (1-\varepsilon) \times 10^2$ and $0.1 \leq \lambda_s/\lambda_f \leq 10$, where λ_f is the thermal conductivity of saturating fluid. A grid resolution 100×100 is employed and $C_0 = (C_f + C_s)/2$.

Figures 4-7 plot the streamlines and isotherms for various λ_s/λ_f. The fluid-saturated porous media becomes cold as λ_s/λ_f decreases. When λ_s/λ_f =

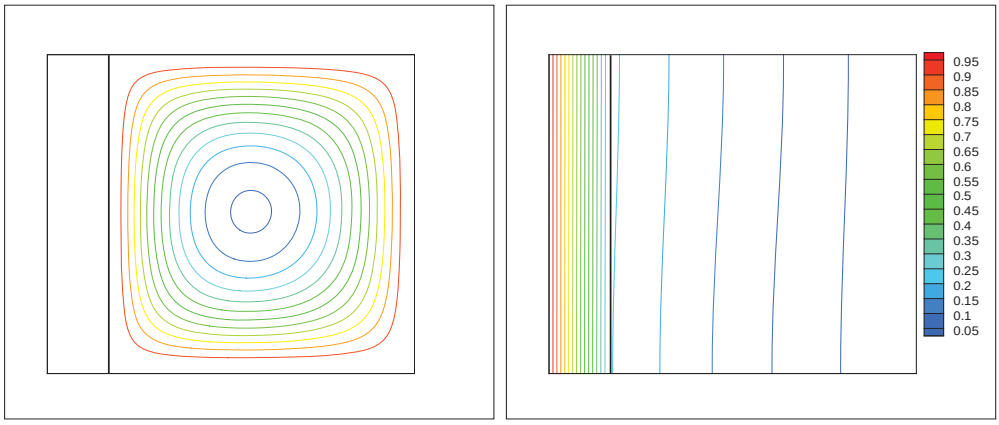


(a) (b)

Fig. 4. (a) streamlines and (b)isotherms for $\lambda_s/\lambda_f = 0.1$.

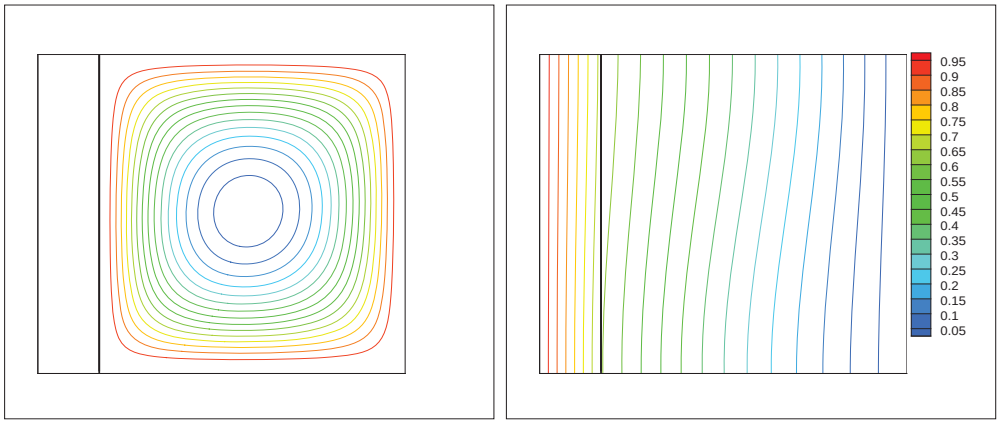
0.1, the solid wall acts as an adiabatic layer so the difference between the temperature on the interface and that on the cold wall is very small, as shown by Fig. 8. For a high λ_s/λ_f, however, the conjugate wall becomes very much conductive. As a result, the temperature on the interface increases with λ_s/λ_f (see Fig. 8), which intensifies the flow circulation in the porous cavity (see Figs. 4-7). Furthermore, it can be observed that with the increase of λ_s/λ_f the isotherms begin to depart from its vertical patter in the porous cavity. Such phenomenon implies the heat transfer mechanism begins changing from a predominant conduction heat transfer regime to convection heat transfer. The above observations are consistent with those indicated by Fig. 6 in Ref.[2].

For a quantitative comparison, the local temperature profile along the interface and the average Nusselt number on the interface are presented by Fig. 8 and Table 1, respectively, together with the data published in Ref. [2]. There is an excellent agreement between them.



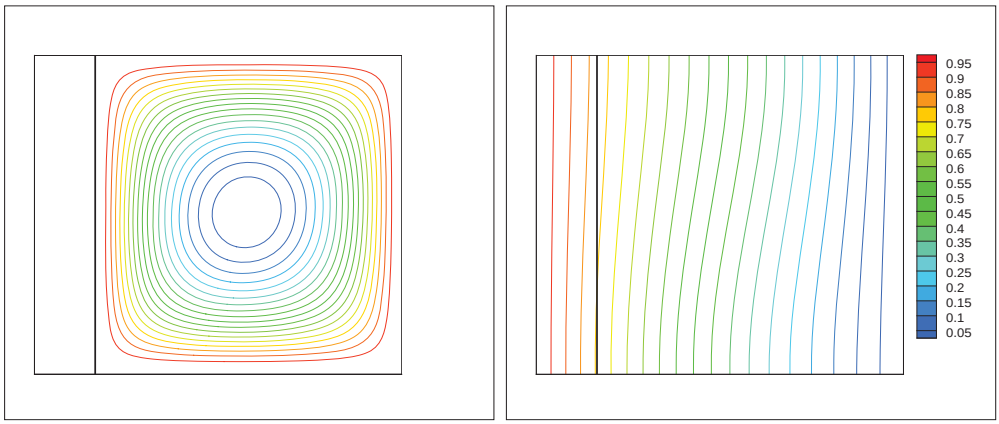
(a) (b)

Fig. 5. (a) streamlines and (b)isotherms for $\lambda_s/\lambda_f = 1$.



(a) (b)

Fig. 6. (a) streamlines and (b)isotherms for $\lambda_s/\lambda_f = 5$.



(a) (b)

Fig. 7. (a) streamlines and (b)isotherms for $\lambda_s/\lambda_f = 10$.

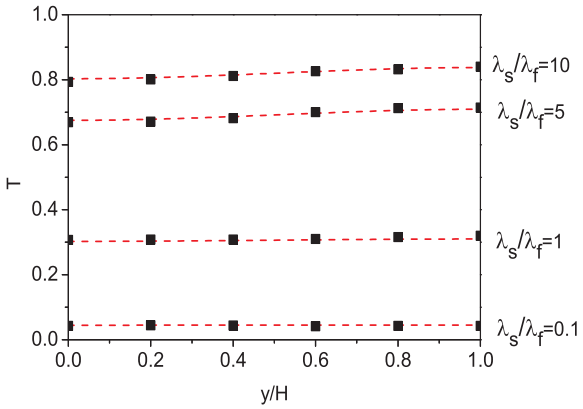


Fig. 8. Temperature profile along the interface: squares-data in Ref.[2], dashed lines-present results.

Table 1
Average Nusselt number on interface.

λ_s/λ_f	0.1	1	5	10
Ref. [2]	0.478	3.433	7.710	9.168
present	0.4897	3.5088	7.9936	9.5366

4.3 Conjugate natural convection in a fluid-saturated porous cavity with a hot triangular thick wall

Figure 10 illustrates the configuration of conjugate natural convection in a fluid-saturated porous cavity with a hot triangular thick wall, which is the same as that in Ref. [3]. In this case, there is an inclined interface. For traditional numerical approaches, a complicated treatment is required for such interface [3,9]. The parameters used in the present research is $C_s = C_f = 1$, $\sigma = 1$, $\varepsilon = 0.99$, $Da = 10^{-3}$, $Pr = 1$, $Ra = 10^4$, $\lambda_s/\lambda_m = 23.8$. The size of the triangular solid wall can change from $0.1 \leq d/H \leq 1$. A grid resolution 100×100 is employed and $C_0 = (C_f + C_s)/2$.

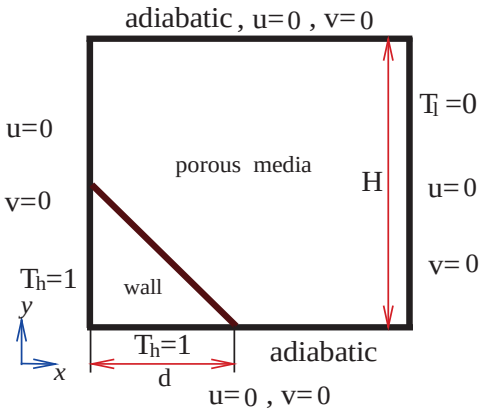
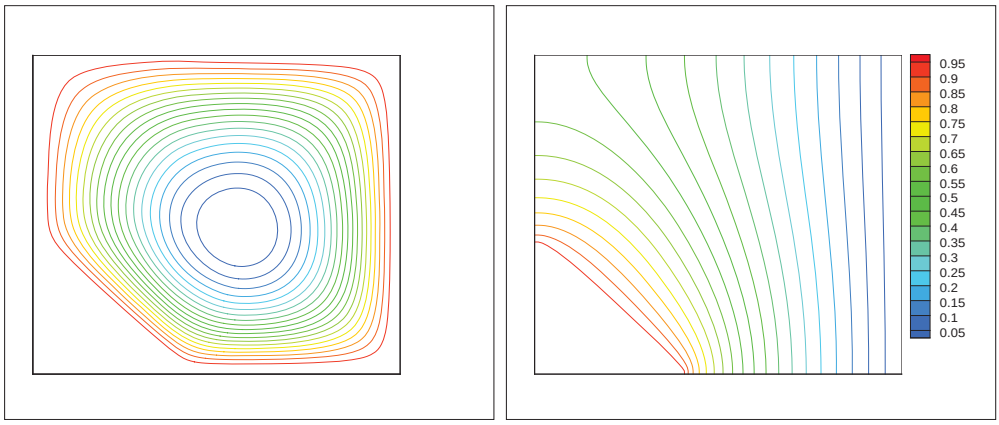


Fig. 9. Schematic configuration of conjugate natural convection in a fluid-saturated porous cavity with a hot triangular thick wall.



(a) (b)

Fig. 10. (a) streamlines and (b)isotherms for $d/H = 0.4$.

Figure 10 depicts the streamlines and isotherms in the cavity when $d/H = 0.4$. The buoyant force induced by the hot triangular solid wall drives a clockwise circulation in the porous cavity. As the buoyant force is weak ($Da \times Ra = 10$), the porous cavity is dominated by heat conduction mechanism. These observations agree well with those illustrated by Fig.4 in Ref. [3].

The average Nusselt number on the inclined interface is listed by Table 2, compared with the data given by Ref. [3]. One can observe that heat transfer

Table 2
Average Nusselt number on interface.

	$d/H = 0.1$	$d/H = 0.4$	$d/H = 1$
Ref. [3]	0.39	0.97	4.46
present	0.4059	0.9819	4.3919

is enhanced by a larger triangular thick wall. The present prediction agrees with that in Ref. [3].

5 Conclusion

Conjugate heat transfer between fluid-saturated porous media and solid walls is commonly found in many practical applications. Unfortunately, for traditional numerical approaches, it is still a great challenge to model conjugated problems with complicated interfaces. To remedy such drawback, in the present work, a new simple approach, with the aid of the intrinsic advantage of the LB method, is proposed. Its feasibility is validated by three nontrivial benchmark tests. Compared with the available LB models for conjugate problems, there is no additional term in the present scheme, which can guarantee that the simplicity, accuracy and stability of the present model are better than previous models.

Although in the present study we only take a single-relaxation-time LB model as an example to show how to address conjugate heat transfer problems between fluid-saturated porous media and solid walls by a simple way, the extension to its multiple-relaxation-time counterpart is straightforward. It will

be considered in our future work as for a single-relaxation-time LB model the accuracy of numerical results will depend on the chosen relaxation time.

Finally, besides the present work, the available numerical approaches [2,3,34] for simulating conjugate heat transfer between fluid-saturated porous media and solid wall all have not been validated by experimental data. Such validation is desired in the future when appropriate experimental data are available.

Acknowledgments

This work has received funding from the Universidad Carlos III de Madrid, the European Unions Seventh Framework Programme for research, technological development and demonstration under grant agreement No. 600371, el Ministerio de Economa y Competitividad (COFUND2014-51509), el Ministerio de Educacin, cultura y Deporte (CEI-15-17) and Banco Santander. S. Chen would also acknowledge the support from the British Newton Alumni Fellowship Scheme.

References

[1] Ingham DB, Bejan A, Mamut E, Pop I. Emerging technologies and techniques in porous media. 2003 Kluwer, London.

[2] Al-Amiri A, Khanafer K, Pop I. Steady-state conjugate natural convection in a

fluid-saturated porous cavity. International Journal of Heat and Mass Transfer 2008;51: 4260-4275

[3] Chamkha AJ, Ismael MA. Conjugate heat transfer in a porous cavity filled with nanofluids and heated by a triangular thick wall. International Journal of Thermal Science 2013;67: 135-151.

[4] Perelman T.L. On conjugated problems of heat transfer. International Journal of Heat and Mass Transfer 1961;3: 293-303.

[5] Dorfman A, Renner Z. Conjugate problems in convective heat transfer: Review. Mathematical Problems in Engineering 2009; 2009: 1-27

[6] Giles MB. Stability analysis of numerical interface conditions in fluidstructure thermal analysis. International journal for numerical methods in fluids 1997;25:421-436

[7] Roux FX, Garaud JD. Domain Decomposition Methodology with Robin Interface Matching Conditions for Solving Strongly Coupled Fluid-Structure Problems. International Journal for Multiscale Computational Engineering 2009;7: 29-38

[8] Meng F, Banks JW, Henshaw WD, Schwendeman DW. A stable and accurate partitioned algorithm for conjugate heat transfer. Journal of Computational Physics 2017;344: 51-85

[9] Sato N, Takeuchi S, Kajishima T, Inagaki M, Horinouchi N. A consistent direct discretization scheme on Cartesian grids for convective and conjugate heat transfer. Journal of Computational Physics 2016;321:76-104

[10] Kang S, Iaccarino G, Ham F. DNS of buoyancy-dominated turbulent flows on a bluff body using the immersed boundary method. Journal of Computational Physics 2009;228:3189-3208

[11] Yu Z, Shao X, Wachs A. A fictitious domain method for particulate flows with heat transfer. Journal of Computational Physics 2006;217: 424-452.

[12] Lin P, Jaluria Y. Conjugate transport in polymer melt flow through extrusion dies. Polymer Engineering and Science. 1997;37:1582-1595, .

[13] Betchen L, Straatman AG, Thompson BE. A nonequilibrium finite-volume model for conjugate fluid/porous/solid domains. Numerical Heat Transfer A, 2006;49:543-65.

[14] Guedes ROC, Ozisik MN, Cotta RM. Conjugated periodic turbulent forced convection in a parallel plate channel. Journal of Heat Transfer 1994;116:40-46,

[15] Vaszi AZ, Elliott L, Ingham DB, Pop I. Conjugate Free Convection above a Heated Finite Horizontal Flat Plate Embedded in a Porous Medium. International Journal of Heat and Mass Transfer 2002;45: 2777-2795

[16] Hung CI, Chen CK, Cheng P. Transient Conjugate Natural Convection Heat Transfer along a Vertical Plate Fin in a High-Porosity Medium. Numerical Heat Transfer, Part A 1989;15:133-148

[17] Baytas AC, Liaqat SA, Grosan T, Pop I. Conjugate Natural Convection in a Square Porous Cavity. Heat and Mass Transfer 2001;37:467-473

[18] Saeid NH. Conjugate Natural Convection in a Porous Enclosure: Effect of Conduction in One of the Vertical Walls. International Journal of Thermal

Science 2007;46:531-539

[19] Hong JS, Kim MC, Choi CK. Onset of Buoyancy-Driven Instability in Porous Media Melted from Below. Transport in Porous Media 2007;67:229-241

[20] Bear J. Dynamics of Fluids in Porous Media. 2013 Elsevier, New York.

[21] Succi S. The lattice Boltzmann equation for fluid dynamics and beyond. Oxford: Oxford university press;2001.

[22] Wu W, Zhang S, Wang S. A novel lattice Boltzmann model for the solid-liquid phase change with the convection heat transfer in the porous media. International Journal of Heat and Mass Transfer 2017;104: 675-687.

[23] Huang Y, Chen Q. A numerical model for transient simulation of porous wicked heat pipes by lattice Boltzmann method. International Journal of Heat and Mass Transfer 2017;105: 270-278

[24] Grucelski A, Pozorski J. Lattice Boltzmann simulations of heat transfer in flow past a cylinder and in simple porous media. International Journal of Heat and Mass Transfer 2015;86: 139-148

[25] Shokouhmand H, Jam F, Salimpour MR. Simulation of laminar flow and convective heat transfer in conduits filled with porous media using Lattice Boltzmann Method. International Communications in Heat and Mass Transfer 2009;36: 378-384

[26] Chen S, Yang B, Zheng C. A lattice Boltzmann model for heat transfer in heterogeneous media. International Journal of Heat and Mass Transfer 2016;102:637-644.

[27] Wang J, Wang M, Li Z. A lattice Boltzmann algorithm for fluidCsolid conjugate heat transfer. International Journal of Thermal Sciences 2007;46: 228-234.

[28] Meng F, Wang M, Li Z. Lattice Boltzmann simulations of conjugate heat transfer in high-frequency oscillating flows. International Journal of Heat and Fluid Flow 2008;29:1203-1210

[29] Li L, Chen C, Mei R, Klausner JF. Conjugate heat and mass transfer in the lattice Boltzmann equation method. Physical Review E 2014;89:043308/1-043308/21.

[30] Le G, Oulaid O, Zhang JF. Counter-extrapolation method for conjugate interfaces in computational heat and mass transfer. Physical Review E 2015;91:033306/1-033306/11

[31] Hu Y, Li D, Shu S, Niu X. Full Eulerian lattice Boltzmann model for conjugate heat transfer. Physical Review E 2015;92:063305/1-063305/12

[32] Karani H, Huber C. Lattice Boltzmann formulation for conjugate heat transfer in heterogeneous media. Physical Review E 2015;91:023304/1-023304/10.

[33] Chen S, Yan YY, Gong W. A simple lattice Boltzmann model for conjugate heat transfer research. International Journal of Heat and Mass Transfer 2017;107: 862-870

[34] Gao D, Chen Z, Chen L, Zhang D. A modified lattice Boltzmann model for conjugate heat transfer in porous media. International Journal of Heat and Mass Transfer 2017;105 673-683

[35] Liu Q, He YL, Li Q, Tao WQ. A multiple-relaxation-time lattice Boltzmann

model for convection heat transfer in porous media. International Journal of Heat and Mass Transfer 2014;73:761-775

[36] Guo Z, Zhao TS. A lattice Boltzmann model for convection heat transfer in porous media. Numerical Heat Transfer, Part B 2005;47:157-177.

[37] Gao D, Chen Z, Chen L. A thermal lattice Boltzmann model for natural convection in porous media under local thermal non-equilibrium conditions. International Journal of Heat and Mass Transfer 2014;70:979-989

[38] Chen S, Luo K, Zheng C. A simple enthalpy-based lattice Boltzmann scheme for complicated thermal systems. Journal of Computational Physics 2012;231:8278-8294.